Adaptive Decentralized Output-Constrained Control of Single-Bus DC Microgrids

Jiangkai Peng, Student Member, IEEE, Bo Fan, Student Member, IEEE, Jiajun Duan, Member, IEEE, Qinmin Yang, Member, IEEE, and Wenxin Liu, Senior Member, IEEE

Abstract—A single-bus DC microgrid can represent a wide range of applications. Control objectives of such systems include high-performance bus voltage regulation and proper load sharing among multiple Distributed Generators (DGs) under various operating conditions. This paper presents a novel decentralized control algorithm that can guarantee both the transient voltage control performance and realize the predefined load sharing percentages. First, the output-constrained control problem is transformed into an equivalent unconstrained one. Second, a two-step backstepping control algorithm is designed based on the transformed model for bus-voltage regulation. Since the overall control effort can be split proportionally and calculated with locally-measurable signals, decentralized load sharing can be realized. The control design requires neither accurate parameters of the output filters nor load measurement. The stability of the transformed systems under the proposed control algorithm can indirectly guarantee the transient bus voltage performance of the original system. Additionally, the high-performance control design is robust, flexible, and reliable. Switch-level simulations under both normal and fault operating conditions demonstrate the effectiveness of the proposed algorithm.

Index Terms—Decentralized control, DC microgrids, paralleled converters, output constraint

I. INTRODUCTION

In recent years, microgrids are gaining popularity due to the penetration of renewable energy sources, the distributed allocation of generation, and the increasing participation of consumers [1]. A microgrid usually consists of multiple Distributed Generators (DGs) such as fuel cells, photovoltaics; distributed energy storages such as batteries, ultra-capacitors; and loads [2-4]. Since the majority of DGs, energy storage devices, and modern electronic loads operate in Direct Current (DC) [5], the formation of DC microgrids is recognized as a simple and natural solution by avoiding additional AC/DC conversion stages. Moreover, a DC microgrid has several advantages over its AC counterpart, e.g. the circumvention of problems with harmonics, unbalances, synchronization, and reactive power flows [6].

One typical DC microgrid topology is derived from connecting multiple converter-interfaced DGs in parallel to a common DC bus, which supplies electric power to the loads [7]. The parallel operation of DGs offers several advantages including expandability, reliability, efficiency, and ease of maintenance [8]. This single-bus topology represents a wide range of applications such as electrical power systems of avionics, automotive, telecom, marine, and rural areas [9-11]. It is a DC microgrid in its simplest form that can be regarded as the building blocks of multiple-bus systems [6]. If such a DC microgrid can be well regulated, the control of large-scale microgrids can be achieved by integrating appropriate secondary and tertiary controllers.

To achieve a safe and efficient operation of DC microgrids with paralleled converters, two control objectives should be realized: voltage regulation and load sharing [1]. Voltage regulation ensures the common DC bus voltage tracking a predefined trajectory under various loading conditions. Load sharing properly shares the load according to the capacities and operating costs of DGs [12]. Many control methods [13-23] have been proposed for such DC microgrids. Existing solutions can be mainly classified into two categories according to the use of communication links [24] as introduced below.

The first type of control solutions requires various schemes of communications to coordinate operations of subsystems. Accordingly, the solutions can be classified into centralized control [14], Master-Slave Control (MSC) [15],[16], Average Load Sharing (ALS) [25] and Circular Chain Control (3C) [17]. These control schemes usually can achieve both control objectives satisfactorily. However, the requirement for communications may lead to other problems, such as increased cost, decreased reliability, and lack of scalability and flexibility [26].

In comparison, the second type of control solutions is communication-free and mainly based on droop control [18-20]. Such solutions linearly adjust the bus voltage references based on predefined droop equations. However, traditional droop control method has some drawbacks. The voltage regulation and load sharing cannot be realized simultaneously. To overcome its limitations, some improvements have been made including nonlinear and adaptive droop control methods [21-23]. However, the tradeoff between voltage regulation and
load sharing still persists due to the simple static proportional rule.

Furthermore, the traditional control solutions can only guarantee convergence but not transient performance during the process. Since DC microgrids are usually converter-based and have much smaller inertia compared to conventional AC ones, disturbances such as sudden load changes may cause abrupt voltage overshoot or dip in transient-state. Such oscillations, even if eventually converge, are not only harmful to sensitive loads but also likely to cause an unexpected false action of protection system [27]. As a result, transient control performance should be guaranteed for high-performance applications.

To address the aforementioned problems, advanced control designs [28] are needed for single-bus DC microgrid. In this paper, a novel adaptive decentralized control scheme is presented. Since the decentralized design does not require communications for bus voltage control and load sharing, flexibility and reliability of the system can be improved. Furthermore, the adaptive design does not require accurate system parameters, which further enhances the robustness. The transient performance is enhanced by introducing output-constrained control technique. During the control design, the original control model with time-varying output constraints is transformed into an unconstrained one. Convergence of the transformed system can further guarantee the transient performance of the original system. The overall control effort (total injected current) for bus voltage control can be realized in a decentralized way and assigned to subsystems according to predefined percentages, which means decentralized load sharing. Since only bounds of systems parameters and load disturbance are used in controller design and stability analysis, the designed controller is robust against parameter uncertainties and load changes. Control implementation considers both normal and fault operating conditions with different sizes of disturbances. Simulation studies with switch-level model demonstrate the effectiveness of the algorithm. By integrating with secondary and tertiary control algorithms, the designed algorithm can be extended to large-scale multiple-bus DC microgrids.

The remaining of this paper is organized as follows. Section II presents the system model and the system transformation. Controller design and stability analysis are discussed in Section III. In Section IV, the performance of the proposed controller is validated through switch-level model simulation with MATLAB/Simulink. Finally, Section V provides summaries and discussion on future research.

II. PROBLEM FORMULATION

The model of a single-bus DC microgrid with parallel converters is presented in this section. Bounded transient performance requirement is formulated as constraints on system output. The constrained system is then transformed into an equivalent unconstrained one through model transformation. The subsequent control design based on the transformed system can guarantee both convergence and bounded transient performance of the original system.

A. System Modeling

In this paper, the single-bus DC microgrid with parallel converters, as shown in Fig. 1, is considered [29]. In this microgrid, n DGs are connected in parallel through LC filters to a common DC bus which supplies power to one or more loads. The system dynamics can be expressed as:

\[
\begin{align*}
\dot{v}_0 &= \sum_{j=1}^{n} i_j - \sum_{j=1}^{n} C_j \dot{v}_j \\
i_j &= -\frac{1}{L_j} v_0 - \frac{R_j}{L_j} i_j + \frac{1}{L_j} v_j, \quad j = 1, 2, ..., n
\end{align*}
\]

where \(v_0\) is the common DC-bus voltage; \(R_j, L_j, C_j\) are the parameters of the \(j^{th}\) L-C filter, respectively; \(i_j\) is the output current of converter \(j\), and \(v_j\) is the output voltage of converter \(j\) that serves as the control input of the system.

There are two control objectives in a DC microgrid, namely, voltage regulation and load sharing.

Voltage regulation aims to achieve a desired DC bus voltage \(v_o\), which is realized by making \(v_o\) track a user-defined trajectory \(v_{ref}\). A good controller should also ensure the transient performance of \(v_o\). More specifically, the voltage should always be maintained within predefined time-varying bounds, i.e.,

\[v_o(t) < v_o(t) < \overline{v}_o(t)\]

holds at all times, where \(v_o(t)\) and \(\overline{v}_o(t)\) are the time-varying lower and upper bounds of \(v_o\), respectively, satisfying \(v_o(t) < v_{ref}(t) < \overline{v}_o(t)\).

Load sharing aims to properly share the load among DGs according to their capacities and system operating cost. As the voltage on the common bus is unique, load sharing among all DGs can be realized by adjusting their output currents \(i_j\) of DGs.

B. System Transformation

To achieve the bounded transient control performance over \(v_o\), the original system with time-varying output constraints described by (1) and (2) can be transformed into an equivalent unconstrained one prior to designing the control strategy. This
is achieved by using the system transformation technique [30]. The principle of this transformation is to build a one-to-one mapping of the constrained variable to an exaggerated variable defined within \((-\infty, \infty)\). According to this principle, the stability of the transformed system is sufficient to ensure both the stability and the time-varying output constraints of the original system.

Firstly, define the system tracking error \(e_v\) as
\[
e_v = v_o - v_{ref}.
\]
(3)

Thereafter, the constraints on \(v_o\) can be rephrased as
\[
e_v < e_v < \bar{e}_v
\]
(4)

where \(e_v = v_o - v_{ref} < 0\) and \(\bar{e}_v = \bar{v}_o - v_{ref} > 0\).

Next, the constrained tracking error \(e_v\) is transformed into an unconstrained variable \(\xi\) using a transformation function \(T(\cdot)\) defined as
\[
\xi = T(e_v, \bar{e}_v, \bar{g}_v).
\]
(5)

This transfer function \(T(\cdot)\) is designed to be smooth, strictly increasing and satisfy
\[
\lim_{e_v \to \bar{e}_v} \xi = +\infty
\]
\[
\lim_{e_v \to -\infty} \xi = -\infty.
\]
(6)

In this paper, the following typical transformation function [30] is chosen for its flexibility:
\[
\xi = T(e_v, \bar{e}_v, \bar{g}_v) = \begin{cases}
 l\left(\tan \frac{\pi \bar{g}_v}{2\bar{e}_v} + \frac{\pi e_v}{2\bar{e}_v}\right) + e_v, & 0 < e_v < \bar{e}_v \\
 0, & e_v = 0 \\
 -l\left(\tan \frac{\pi \bar{g}_v}{2\bar{e}_v} - \frac{\pi e_v}{2\bar{e}_v}\right) + e_v, & \bar{g}_v < e_v < 0
\end{cases}
\]
(7)

where \(l\) is a user-defined positive constant.

It is evident that the transformation function in (7) satisfies the requirements in (6). Hence, as long as \(\xi\) exists, the constraint on the original system tracking error in (4) holds.

Notice that the transformation function (7) is differentiable on \(e_v \in (\bar{g}_v, \bar{e}_v)\), taking its time derivative gives
\[
\dot{\xi} = a\dot{e}_v + b
\]
(8)

where
\[
a = \begin{cases}
 \frac{\pi}{2} + 1, & 0 < e_v < \bar{e}_v \\
 1, & e_v = 0 \\
 \frac{\pi}{2} + 1, & \bar{g}_v < e_v < 0
\end{cases}
\]
(9)
\[
b = \begin{cases}
 -\frac{\pi \bar{g}_v}{2\bar{e}_v} - \frac{\pi e_v}{2\bar{e}_v}, & 0 < e_v < \bar{e}_v \\
 0, & e_v = 0 \\
 -\frac{\pi \bar{g}_v}{2\bar{e}_v} + \frac{\pi e_v}{2\bar{e}_v}, & \bar{g}_v < e_v < 0
\end{cases}
\]
(10)

and \(\bar{a} = \frac{l}{\cos \frac{\pi \bar{g}_v}{2\bar{e}_v}}\) - \(l\), \(\bar{a} = \frac{l}{\cos \frac{\pi \bar{g}_v}{2\bar{e}_v}} + l\).

Combining (1) and (8) yields the transformed system dynamic equations
\[
\begin{bmatrix}
\dot{\xi} = a\dot{e}_v + b
\end{bmatrix}
\]
(11)

\[
\begin{cases}
 i_j = -\frac{1}{L_j}v_o - \frac{R_j}{L_j}i_j + \frac{1}{L_j}v_j, & j = 1, 2, ..., n
\end{cases}
\]
(12)

where \(d = i_{load} - \frac{b}{a}\sum_{j=1}^{n}C_j\) is considered as the disturbance of the transformed system.

Consequently, the original system with time-varying output constraints is transformed into an equivalent unconstrained one, whose controller design and stability analysis will be introduced in the following section.

### III. CONTROL DESIGN AND IMPLEMENTATION

In this section, the decentralized control law is formulated using the transformed system dynamics (11) based on the backstepping technique [31]. Output filter parameters \((R_j, L_j, C_j)\) are considered to be unknown but bounded to improve the controller’s robustness against parameter uncertainties. Voltage regulation and load sharing can be achieved simultaneously, and the system stability is demonstrated through rigorous Lyapunov synthesis.

Before proceeding, the following reasonable assumptions are introduced:

**Assumption 1**: the output filter parameters are unknown but bounded, i.e.,
\[
\begin{align*}
R & \leq R_j \leq \bar{R} \\
L & \leq L_j \leq \bar{L} \\
C & \leq C_j \leq \bar{C}
\end{align*}
\]

with \(\bar{R}, \bar{R}_j, \bar{L}_j, \bar{C}_j, \bar{C}\) being known positive constants.

**Assumption 2**: the disturbance \(d\) and its first time derivative \(d\) are unknown but bounded, i.e.,
\[
|d| \leq D_0, |\dot{d}| \leq D_1
\]

where \(D_0, D_1\) are known positive constants.

#### A. Decentralized Controller Design

Notice that the transformed system dynamics described by (11) is in strict-feedback (lower triangular) form. The backstepping principle can be applied to recursively stabilize the system in two steps. In the first step, \(i_j\) is considered as the virtual control input to stabilize \(\xi\) and achieve proper load sharing. In the second step, \(v_j\) is the actual control input to stabilize \(i_j\). Hence, the overall system stability can be guaranteed.

**Step 1: Design of the virtual control input \(i_j\)**

Consider the following Lyapunov candidate
\[
V_1 = \sum_{j=1}^{n}C_j \xi^2.
\]
(12)

Substituting the first differential equation in (11) into its time derivative yields
\[
\dot{V}_1 = a\xi \left( \sum_{j=1}^{n}i_j - d \right).
\]
(13)

Based on (13), a desired virtual control input \(i_j^*\) can be designed as
where \( k_p \) is a positive constant, \( p_j \) is the portion of the total output current that converter \( j \) supplies, satisfying \( \sum_{j=1}^{n} p_j = 1 \). This proportion \( p_j \) is able to regulate the output current of converter \( j \) thus achieving proportional load sharing, \( \hat{d} \) is the estimation of the disturbance \( d \), whose update law can be designed as follows
\[
\hat{d} = \begin{cases} 
-\gamma_t a\xi, & \text{if } |\hat{d}| < D_0, \text{ or } |\hat{d}| = D_0 \text{ and } d\xi \geq 0 \\
0, & \text{if } |\hat{d}| = D_0 \text{ and } d\xi < 0 
\end{cases}
\] (15)

where \( \gamma_t \) is a positive adaption gain.

Now denote the tracking error \( e_{ij} \) of the virtual control, and estimation error \( \hat{d} \) of the disturbance, which are given as
\[
e_{ij} = i_j - i_j^*, \tag{16}
\]
\[
\hat{d} = d - \hat{d}. \tag{17}
\]

Combining (13), (14), (16), (17) yields
\[
\dot{V}_1 = -k_p a\xi^2 - a\xi \dot{d} + a\xi \sum_{j=1}^{n} e_{ij}. \tag{18}
\]

Further, define the following Lyapunov function \( V_2 \)
\[
V_2 = V_1 + \frac{1}{2\gamma_t} \hat{d}^2. \tag{19}
\]

Substituting (18) into its time derivative yields
\[
\dot{V}_2 = -k_p a\xi^2 - a\xi \frac{d}{y_L} + a\xi \sum_{j=1}^{n} e_{ij} + \frac{2D_0D_1}{\gamma_L} + \frac{2D_0D_1}{\gamma_L} + 2nk_p\varepsilon. \tag{20}
\]

According to Assumption 2, (20) becomes
\[
\dot{V}_2 \leq -k_p a\xi^2 + a\xi \sum_{j=1}^{n} e_{ij} + 2D_0D_1 + 2nk_p\varepsilon. \tag{21}
\]

Next, substituting (15) into (21) yields
\[
\dot{V}_2 \leq -k_p a\xi^2 + a\xi \sum_{j=1}^{n} e_{ij} + \frac{2D_0D_1}{\gamma_L} + 2nk_p\varepsilon. \tag{22}
\]

**Step 2: Design of the actual control input \( v_j \)**

By definition formula (16) of \( e_{ij} \), the time derivative of \( e_{ij} \) is
\[
\dot{e}_{ij} = -\frac{1}{L_j} v_o - \frac{R_j}{L_j} i_j + \frac{1}{L_j} v_j - i_j. \tag{23}
\]

Consider the following augmented Lyapunov function \( V_3 \)
\[
V_3 = V_2 + \frac{1}{2} \sum_{j=1}^{n} L_j e_{ij}, \tag{24}
\]

Recalling (22), (23), and taking its time derivative gives
\[
\dot{V}_3 \leq -k_p a\xi^2 + \frac{2D_0D_1}{\gamma_L} + \frac{2D_0D_1}{\gamma_L} + 2nk_p\varepsilon. \tag{25}
\]

Based on (25), the actual control input \( v_j \) can be designed as
\[
v_j = -k_{ij} e_{ij} - a\xi + v_o - \overline{R}_i j \tanh \left( \frac{e_{ij}\overline{R}_i j}{\varepsilon} \right) - L_i j \tanh \left( \frac{e_{ij} L_i j}{\varepsilon} \right) \tag{26}
\]

where \( \varepsilon, k_{ij} \) are user-defined positive constants.

Substituting (26) into the time derivative of \( V_3 \) yields
\[
\dot{V}_3 \leq -k_p a\xi^2 - \sum_{j=1}^{n} k_{ij} e_{ij}^2 + \frac{2D_0D_1}{\gamma_L} + \frac{2D_0D_1}{\gamma_L} + 2nk_p\varepsilon. \tag{27}
\]

According to the following inequality \([32]\)
\[
0 \leq |\eta| - \eta \tanh \left( \frac{D_0}{y_L} \right) \leq k_p\varepsilon \tag{28}
\]

where \( k_p \) is a constant satisfying \( k_p = e^{-(k_p+1)} \), i.e. \( k_p = 0.2758 \). Then (27) can be written as
\[
\dot{V}_3 \leq -k_p a\xi^2 - \sum_{j=1}^{n} k_{ij} e_{ij}^2 + \frac{2D_0D_1}{\gamma_L} + 2nk_p\varepsilon. \tag{29}
\]

It is evident that the designed control \( v_j \) in (26) only utilizes locally-measurable signals \( (v_o, i_j) \), which implies that the controller is fully decentralized. Furthermore, bounds of output filter parameters \( (\overline{R}, \overline{L}, \overline{C}) \) are used instead of their actual values, which ensures that the control method is robust against system parameter uncertainties.

**B. Stability Analysis**

The main theoretical results for the proposed control strategy are illustrated in the following theorem.

**Theorem 1:** Consider the converter-interfaced single bus DC microgrid characterized by (1), satisfying Assumption 1-2, and the control law (26). Then the following facts hold: 1) All signals in the closed-loop system are bounded; 2) Voltage regulation (convergence and output constraint) and load sharing can be achieved. The voltage tracking error \( e_o \) and the current error \( e_{ij} \) can be made arbitrarily small by tuning the gains \( k_p, k_{ij} \) and \( \gamma_t \).

**Proof:** Consider the following augmented Lyapunov function \( V_3 \)
\[
V_3 = \frac{\sum_{j=1}^{n} C_j}{2} \xi^2 + \frac{1}{2\gamma_L} \hat{d}^2 + \frac{\sum_{j=1}^{n} L_j e_{ij}^2}{2}. \tag{30}
\]

Its time derivative \( \dot{V}_3(t) \) (27) can be rewritten in the following form
\[
\dot{V}_3(t) \leq -c_1 V_3(t) + c_2 \tag{31}
\]

where
\[
c_1 = \min \left\{ \frac{2k_p a}{\gamma_L}, \frac{2k_{ij}}{L_j}, \ldots, \frac{2k_{ij}}{L_n} \right\}, \tag{31}
\]
\[
c_2 = \frac{2D_0D_1}{\gamma_L} + 2nk_p\varepsilon \tag{32}
\]

Then one can conclude that all the signals in the closed-loop system (11) are bounded via Lyapunov synthesis [33]. Furthermore, the proposed control law (26) guarantees that [33]
\[
\lim_{t \to \infty} V(t) \leq \frac{c_2}{c_1} \tag{33}
\]

This implies that the transformed voltage error \( \xi \) and current error \( e_{ij} \) can be made arbitrarily small by appropriate tuning of
gains $k_v$, $k_{ij}$ and $\gamma_L$. Combined with the definition of system transformation, the original voltage tracking error $e_v$ is guaranteed to stay within the output constraint. Thus, voltage regulation and load sharing can be achieved.

C. Controller Implementation

The block diagram of the proposed control design is shown in Fig. 2. For each DG, signals of the original system are converted to corresponding signals of the transformed system for control signal calculation. Then, the desired output current $i_j^*$ is calculated. Finally, the calculated actual control signal $v_j$ is compared with a sawtooth waveform to generate pulse width modulation (PWM) signals for converter control. Since only local signals are needed for control signal calculation, there is no need for inter-subsystem communication and the proposed control algorithm is eventually decentralized.

Another crucial aspect of controller implementation is bound setting. Above analysis shows that bounded transient voltage response can be guaranteed in ideal conditions. In practice, the voltage response is limited by many factors of the physical system such as switching frequency, maximum achievable voltage of the converter and controller sampling frequency, etc. Consequently, disturbances such as a large step load change, can make the output voltage jump beyond its bounds. It is evident that the controller will fail as the system transformation cannot be performed properly in this case. Thus, the transient bounds on $v_j$ should be dynamically adjusted to balance controllability and control performance. During controller implementation, two operating conditions are considered, i.e. normal and abnormal operating conditions. The designed bound setting strategy is introduced below and illustrated in Fig. 3.

For normal operating condition, the voltage deviation is maintained within 10% of nominal voltage ($|e_v|<10\%V_{ref}$). If voltage deviation is smaller than 5% of nominal voltage ($|e_v|<5\%V_{ref}$), the system is considered working under small disturbances and the bounds are set to constants $\pm 5\%V_{ref}$. Such setting avoids unnecessary voltage bounds’ updating that may cause extra disturbance to the system. If the voltage deviation exceeds 5% but is smaller than 10% of the nominal voltage ($5\%V_{ref}<|e_v|<10\%V_{ref}$), the system is considered under large disturbance, the bounds are increased to $\pm 10\%V_{ref}$, then quickly converge back to original bounds of $\pm 5\%V_{ref}$ following the modified power function below:

$$
\bar{e}_v = -e_v = \frac{5\%V_{ref}}{10^7 \times (t - t^*)^3 + 1} + 5\%V_{ref}
$$

(34)

where $t^*$ is the most recent instant of time large disturbance is detected.

If voltage deviation exceeds the transient bounds under a new large disturbance, $t^*$ will be updated and the bounds will be reset as shown in Fig. 4.

If the voltage deviation exceeds 10% of the nominal voltage ($|e_v|<10\%V_{ref}$), the system is considered as working in abnormal or fault operating conditions. Once such conditions are detected, the bound function for voltage deviation will be set according to (35).

$$
\bar{e}_v = -e_v = \frac{95\%V_{ref}}{10^6 \times (t - t^*)^3 + 1} + 5\%V_{ref}.
$$

(35)

Since the initial bound ($\pm 100\%V_{ref}$) is very large, controllability can be maintained even under serious fault that grounds the bus voltage. The relaxed bound setting allows protection system to react so as to clear the fault. Once the fault is cleared, the voltage deviation will eventually converge into the constant bounds of $\pm 5\%V_{ref}$. The concept of dynamic bound setting is further illustrated through simulation studies in next section.

IV. CASE STUDY

The proposed control scheme is simulated on a single bus DC microgrid with 3 parallel-DGs as shown in Fig. 1. The simulations are conducted on a detailed switch-level model using Simscape Power System toolbox of MATLAB/Simulink. The proposed controller is tested under both normal and abnormal operating conditions.
A. System Definition

The single bus DC microgrid has 3 parallel-connected converters. The converters all operate at a switching frequency of 10 kHz. The controller sampling period is set to 100 μs. Output filter parameters of each converter are different to challenge the control algorithm. For test under the normal operating condition, two sudden load changes are simulated at 0.1 s and 0.3 s. For test under the abnormal operating condition, a short circuit fault is simulated at 0.1 s. The total simulation time is 0.5 s. System parameters are given in TABLE I and controller parameters are given in TABLE II.

In order to better evaluate the performance of the proposed controller, a conventional ALS controller proposed in [25] is implemented for comparison. This ALS controller calculates the output current reference \( i_j^* \) based on total output current \( \sum_{j=1}^{N} i_j \) and load sharing percentage \( p_j \), i.e., \( i_j^* = p_j \sum_{j=1}^{N} i_j \). The resulting current-sharing error signal is then amplified and injected into the inner PI-based voltage and current loops to achieve voltage regulation and proper load sharing simultaneously. It should be noted that communications are required for total output current measurement and control signal delivery. The control parameters of the ALS controller are given in TABLE III.

B. Case I: Even Load Sharing

In this case, the total load is evenly shared among DGs. According to TABLE I, three converters should evenly share the total load current of 6 A before 0.1 s; 8 A from 0.1 s to 0.3 s; and 7.38 A after 0.3 s. Therefore, each DG should generate an output current of 2 A before 0.1 s; 2.67 A from 0.1 s to 0.3 s; and 2.46 A after 0.3 s.

The system responses under the ALS and the proposed controllers are shown in Fig. 4-6.

As demonstrated in Fig. 4, both the ALS controller and the proposed one can achieve voltage regulation during steady-state with a maximum deviation of 0.12 V (0.25%). Furthermore, the larger load change at 0.1 s has triggered the time-varying bounds to increase to \( \pm4.8 \) V while for the smaller disturbance at 0.3 s, the bounds are kept at constant values of \( \pm2.4 \) V. For the proposed controller, the output voltage tracking error \( e_V \) is kept strictly within the transient bound for all time with maximum deviation of 4.29 V at 0.1 s and 1.34 V at 0.3 s. As for the ALS controller, it is evident that it has limited transient performance. The maximum deviation reaches 6.31 V at 0.1 s under large disturbance. For the smaller disturbance at 0.3 s, even though the overshoot deviation of 2.00 V is within the bounds, its amplitude is much higher than that of the proposed controller.

Fig. 5 shows the output current \( i_j \) of each converter, current transitions during transient-state are relatively smooth for both controllers. Their small differences indicate good load sharing. Both controllers have comparable performances in terms of load sharing.
Fig. 6. Control signal $v_j$ under even load sharing condition: (a) ALS controller; (b) proposed controller. 

Fig. 6 shows the control signal $v_j$. It can be observed that the purposed controller has stronger control efforts than that of the ALS controller during transient-state, but they can still be realized by modern converters.

C. Case II: Proportional Load Sharing

In this case, loads are proportionally shared among DGs. The capacities of the three DGs are assumed to be 25%, 25%, and 50%. Therefore, the output currents of each converter should be 1.5 A, 1.5 A, 3 A before 0.1 s; 2 A, 2 A, 4 A from 0.1 s to 0.3 s; and 1.85 A, 1.85 A, 3.69 A after 0.3 s.

The system responses of the ALS controller and the proposed one are shown in Fig. 7-9.

Again, the ALS controller cannot provide a satisfactory transient performance during the serious load changes, whereas the voltage tracking error is kept strictly within the bounds by the proposed controller as demonstrated in Fig. 7.

Furthermore, the output current $i_j$ of each converter is accurately controlled to achieve proportional load sharing, as shown in Fig. 8. The controller signals comparison is shown in Fig. 9.

D. Case III: Short Circuit Fault

In this case, the proposed control strategy is tested under the extreme condition of short circuit fault. According to TABLE I, loads are evenly shared among DGs with a load of 8 Ω before the fault. A short circuit fault at the DC bus is simulated at 0.1 s by decreasing the load resistance to 0.1 Ω. The fault lasts for 8 ms and is cleared by protection system. After the fault, the load resistance becomes 10 Ω. The simulation result is presented in Fig. 10.

As shown in Fig. 10, the short-circuit fault at 0.1 s has caused the output voltage to drop rapidly. When the fault is cleared at 0.108 s, the output voltage quickly recovers. During the process, the output bounds are triggered twice, the first one is

![Fig. 7](image7.png)

Fig. 7. Output voltage tracking error $e_V$ under proportional load sharing condition.

![Fig. 8](image8.png)

Fig. 8. Output current $i_j$ under proportional load sharing condition: (a) ALS controller; (b) proposed controller.

![Fig. 9](image9.png)

Fig. 9. Controller output $v_j$ under proportional load sharing condition: (a) ALS controller; (b) proposed controller.

![Fig. 10](image10.png)

Fig. 10. Output voltage tracking error $e_V$ under short circuit fault.
triggered when the fault happens and the second one is triggered when the fault is cleared. This makes sure that the voltage deviation does not exceed the bounds due to the faults and the voltage is recovered after the fault.

V. CONCLUSIONS

This paper presents a decentralized output-constrained robust control method for single-bus DC microgrids with paralleled converters. Control objectives are to achieve high-performance voltage regulation with bounded transient response and proportional load sharing. By introducing time-varying bounds on the DC bus voltage, the proposed control method can achieve not only convergence but also guarantee bounded transient response. Load sharing is realized in a decentralized fashion with a well-designed adaptive law. Furthermore, the proposed control scheme is robust against system parameter uncertainties as the exact system parameters are not required by the controller. The stability of the closed-loop system is demonstrated through rigorous Lyapunov synthesis. Detailed switch-level model simulations validate the controller's performance under both normal and fault conditions.

REFERENCES


**Jiangkai Peng** (S’2016) received his B.Eng. (Hons) degrees in Electronics and Electrical Engineering from The University of Edinburgh, Edinburgh, UK, and South China University of Technology, Guangzhou, China in 2016. Currently, he is pursuing the Ph.D. degree in Electrical Engineering at Lehigh University, Bethlehem, PA, USA. His research interest includes microgrid, power electronics control system, and power system.

**Bo Fan** (S’15) received the bachelor’s degree in Automation from Zhejiang University, Hangzhou, China, in 2014, where he is currently pursuing the Ph.D. degree in Control Science and Engineering. He is a member of the Group of Networked Sensing and Control (IIPC-NeSC), State Key Laboratory of Industrial Control Technology, Zhejiang University. His current research interests include distributed control, nonlinear systems, and renewable energy systems.

**Jiajun Duan** (S’14) was born in Lanzhou, China, in 1990. He received his B.S. degree in Power system and its automation from Sichuan University, Chengdu, China, and M.S. degree in Electrical Engineering at Lehigh University, Bethlehem, PA in 2013 and 2015, respectively, and the Ph.D. degree in Electrical Engineering from Lehigh University in 2018. Currently, he is a postdoc researcher in GEIRINA, San Jose, CA, USA. His research interest includes power system, power electronics, control systems, machine learning and deep learning.

**Qinmin Yang** (M’10) received the Bachelor’s degree in Electrical Engineering from Civil Aviation University of China, Tianjin, China in 2001, the Master of Science Degree in Control Science and Engineering from Institute of Automation, Chinese Academy of Sciences, Beijing, China in 2004, and the Ph.D. degree in Electrical Engineering from the University of Missouri-Rolla, MO USA, in 2007. From 2007 to 2008, he was a Post-doctoral Research Associate at University of Missouri-Rolla. In 2008, he was a system engineer with Caterpillar Inc. From 2009 to 2010, he was a Post-doctoral Research Associate at University of Connecticut. Since 2010, he has been with the State Key Laboratory of Industrial Control Technology, the College of Control Science and Engineering, Zhejiang University, China, where he is currently a professor. His research interests include intelligent control, renewable energy systems, smart grid, and industrial big data.

**Wenxin Liu** (S’01 – M’05 – SM’14) received the B.S. degree in industrial automation and the M.S. degree in control theory and applications from Northeastern University, Shenyang, China, in 1996 and 2000, respectively, and the Ph.D. degree in electrical engineering from the Missouri University of Science and Technology (formerly University of Missouri–Rolla), Rolla, MO, USA, in 2005. From 2005 to 2009, he was an Assistant Scholar Scientist with the Center for Advanced Power Systems, Florida State University, Tallahassee, FL, USA. From 2009 to 2014, he was an Assistant Professor with the Klipsch School of Electrical and Computer Engineering, New Mexico State University, Las Cruces, NM, USA. He is currently an Associate Professor with the Department of Electrical and Computer Engineering, Lehigh University, Bethlehem, PA, USA. His research interests include power systems, power electronics, and controls. Dr. Liu is an Editor of the IEEE TRANSACTIONS ON SMART GRID, an Associate Editor of the IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, and an Editor of the Journal of Electrical Engineering & Technology.