An Iteration Method for Optimal Energy Flow of Combined Heating and Electricity System

Xiaosong Gao
Smart Grid Key Laboratory of Shaanxi Province, Xi’an Jiaotong University
Xi’an, China
gxs1111@stu.xjtu.edu.cn

Gengfeng Li
Smart Grid Key Laboratory of Shaanxi Province, Xi’an Jiaotong University
Xi’an, China
gengfengli@xjtu.edu.cn

Xiaohu Zhang
Global Energy Interconnection Research Institute North America
San Jose, USA
xiaohu.zhang@geirina.net

Di Shi
Global Energy Interconnection Research Institute North America
San Jose, USA
di.shi@geirina.net

Xu Wang
Smart Grid Key Laboratory of Shaanxi Province, Xi’an Jiaotong University
Xi’an, China
wangxu12@stu.xjtu.edu.cn

Zhaohong Bie
Smart Grid Key Laboratory of Shaanxi Province, Xi’an Jiaotong University
Xi’an, China
zhbie@mail.xjtu.edu.cn

Abstract—In this paper, an iteration method was proposed for calculating the optimal energy flow in a combined heating and electricity system. A basic model of the combined heating and electricity network which consists of electrical distribution network, heating network and combined heating and power units (CHP) was presented firstly. The nonlinear and nonconvex constraints of heating network make the optimal energy flow hard to solve, thus a piecewise linear approximation method is applied to simplify the model, and then, an iteration method based on the mixed integer linear programming was proposed. By considering the temperature difference in heating network energy flow calculation, the proposed iteration method make the results more accurate. Finally, a case study conducted on a 66 bus combined heating and electricity system to demonstrate the effectiveness and superiority of the proposed method.

Keywords—optimal energy flow, iteration method, combined heating and electricity system, mixed integer linear programming, energy flow calculation

I. INTRODUCTION

With the rapid development of economy, the problem between energy and environment has become increasingly prominent. Promoting the absorption of renewable energy and improving the energy supply efficiency is becoming more and more important. The integrated energy system is a new kind of energy system [1]-[2], which usually combines the electricity, heat, gas and other systems, thus, by the complementary of various energies, it can improve the overall efficiency of the energy system, and increase the flexibility to balance fluctuations of the renewable energy. The coupling of electric energy and heat energy is an important form of energy coupling and has been widely used in various countries. In a combined heating and electricity system, the two networks are linked closely through the combined heat and power (CHP) units. Compared with the independent optimization of the heating network and the electrical network, the joint optimization can make the joint system run in the optimal status of economy and security, because synergies between different energy systems are taken into account. The calculation of optimal energy flow is the base of collaborative planning and schedule of combined heating and electricity system, and researches on this topic is of significance.

At present, researches on the optimal power flow model of electricity network has been relatively maturity, thus, more attention are putting into optimal energy flow of combined heating and electricity system. Ref. [3] proposed a modeling method for heating network, but it is too simple to describe the characteristics of actual heating network well. Refs. [4]-[5] proposed a complete model of heating network divided into hydraulic model and thermal model. Based on Newton-Raphson method, hybrid energy flow of combined heating and electricity system is calculated in [4]-[5].

There is some research on methods of optimal energy flow of the combined heating and electricity system at present. Ref. [6] proposes a method for steady-state hybrid energy flow calculation and optimization of the integrated energy systems. Coupling elements among multiple energy systems are modeled by energy hubs. By the concept of energy hubs, the input power of electricity network, natural gas network and heating network is converted into the output electricity and heat through efficiency coupling matrix to analyze and optimize hybrid energy flow. Ref. [7] proposed an optimal allocation method of CHP-based distribution generation in order to maximize the energy supply ability based on the operation characteristic of CHP and distribution network. But the study didn’t consider multiple energy flows constraints. Refs. [8]-[9] solve the problem by intelligent algorithms. However, intelligent algorithm is an algorithm based on intuition or experience. It is easily trapped in the local optimal solution and cannot get the global optimal solution. Ref. [10] used the piecewise linear approximation method to convert the nonlinear and nonconvex model of heating network to the mixed integer linear model. But this method is based on the assumption that the supply temperature at each load node of heating network equals to the supply temperature at heat source node, this obviously is not consistent with the actual situation because of heat loss of pipelines.

On the basis of existing researches, a more accuracy iteration method of solving the optimal energy flow in the combined heating and electricity system is proposed in this paper. The model of combined heating and electricity network which consists of heating network, electricity network and coupling element between these two networks is introduced firstly. Secondly, the nonlinear and nonconvex model of heating network is simplified by the piecewise linear approximation method, which convert the model to a mixed...
integer linear model. In this model, the supply temperature at each node and the return temperature at each source node are assumed to be a specified value, and have difference because of heat loss. Next, an iteration method is proposed to solve this model considering temperature difference in heating network. In each iteration, a heat energy flow is calculated based on the results of the mixed integer linear programming, and the temperature of nodes in the heating network can be obtained after the calculation, which will be given to next solution of the mixed integer linear programming. These processes will continue until the iteration converges. Finally, a case study is conducted to prove the effectiveness and benefit of the proposed method.

The organization of this paper is as follows: The nonlinear and nonconvex basic model of combined heating and electricity system is introduced in Section II. The linear optimal energy flow formulation is addressed in Section III. The iteration method for optimal energy flow is developed in Section IV. A case study based on a 66 bus combined heating and electricity system is introduced to demonstrate the proposed method in Section V. Conclusion of this paper is given in Section VI.

II. MODEL OF COMBINED HEATING AND ELECTRICITY SYSTEM

The schematic diagram of combined heating and electricity system is shown in Fig. 1. Combined heating and electricity system consists of heating network, electricity network and coupling elements between these two networks. The heating network involved in this paper is the district heating network. The electricity network involved in this paper is the distribution network. The role of coupling element is to achieve the conversion between different types of energy. It is the bridge between two subsystems. The coupling element involved in this paper is combined heat and power units (CHP). The CHP is the source of heating network and electricity network.

Because of interdependent energy flow of two networks, the energy supplied by sources can be adjusted according to the price of energy and the distribution of load to minimize the operation cost. So the objective function of the optimal energy flow is presented in (1).

\[ \min F = \sum_{i \in \text{PGS}} a_i P_{G,i,j}^j + \sum_{j \in \text{CHPS}} b_{P,G,j} + c_{j} \Phi_{G,j} \]  

(1)

Where PGS is the set of external power grid sources; CHPS is the set of CHPs; \( P_{G,i} \) is the electricity power from source; \( \Phi_{G,j} \) is the heat power from source; \( a_i \) is the price of electricity power from external power grid sources; \( b_j \) and \( c_j \) are respectively the price of electricity power and heat power from CHPs.

The models of electricity network, heating network and coupling elements are introduced in the following.

A. Electricity Network

The DC power flow is used to establish the model of electricity network in this paper. It is given as (2)-(3).

\[ P_{G,j} - P_{L,j} - B \theta = 0, \forall i \in \text{EB} \]  

(2)

\[ |p_{i} - \theta / x_{i}| \leq p_{i}^\max, \forall k \in \text{EL} \]  

(3)

Where \( P_{L,j} \) denotes electricity power load of bus \( i \); EB denotes the set of buses in the electricity network; \( B_j \) denotes the \( i \) th row of the admittance matrix of electricity branches; \( \theta \) denotes the vector of bus angle; EL denotes the set of transmission lines in the electricity network; \( p_{i} \) denotes the power flow of transmission line \( k \) which isn’t allowed to exceed the upper limit; \( x_{i} \) denotes the reactance of transmission line \( k \).

B. Heating Network

The district heating network consists of the source, the load, the supply network and the return network [4]-[5]. The heat is transferred in the form of hot water or hot steam (hot water in this paper). After the hot water is heated in the source, it flows to the load through the supply network. After the hot water flows into the load, the heat is extracted from the hot water through the radiator, and the temperature of the hot water decreases. The hot water which has been cooled flows back to the source through the return network for the next cycle.

The model of the heating network includes the hydraulic model and the thermal model [4]. The hydraulic model is used to describe the characteristics of the process of hot water flowing in pipelines. The thermal model is used to describe the relationship between the nodal thermal power and temperature, and the mass flow rate of the hot water in the pipeline.

1) Hydraulic model

The continuity of flow can be expressed as (4).

\[ A_{h,i} m = m_{h,i}, \forall i \in \text{HB} \]  

(4)

Where \( A_{h,i} \) denotes the \( i \) th row of the heating network incidence matrix that relates the nodes to the branches; \( m \) denotes the vector of the mass flow rate of pipes; \( m_{h,i} \) denotes the mass flow rate through the \( i \) th node, which is discharged from source or injected to load; HB denotes the set of nodes in heating network.

The mass flow rate of a pipe depends on the pressure of nodes at both ends of pipeline [4]. The pressure of nodes isn’t allowed to exceed the upper limit and lower limit. The above characteristics are described by (5)-(6).

\[ K_{i} m_{h,i} = p_{i} - p_{j}, \forall i \in \text{HL} \]  

(5)

\[ p_{i}^\min \leq p_{i} \leq p_{i}^\max, \forall i \in \text{HB} \]  

(6)

Fig. 1. Schematic diagram of combined heating and electricity system.
Where $K_i$ denotes the resistance coefficients of the $i$th pipe, its calculating method is introduced in detail in [4]; $p$ denotes the pressure of nodes; HL denotes the set of pipelines in heating network.

2) Thermal model

There are three variables describing the temperature state of each node [4]. The supply temperature $T_s$ indicates the temperature of hot water before injected into the load node or discharged from source in supply heating network; the outlet temperature $T_o$ indicates the temperature of hot water flowing out of the load node or injecting into source; the return temperature $T_r$ indicates the temperature of the return water flowing out of the load node after mixing with the water of other pipes in return heating network. Especially, if there is no mix at a node, the outlet temperature equals to the return temperature. Usually, the supply temperature of source and the outlet temperature of load are assumed to be known. The heat power consumed by nodes is calculated as (7) [11]. If a node is source node, the heat power $\Phi$ is negative.

$$\Phi_i = C_p m_i (T_{s,i} - T_{o,i}) \forall i \in HB \quad (7)$$

Where $C_p$ denotes the specific heat of water.

There is heat loss during the heat power transportation. The relationship between the temperature of the end of the pipe and the temperature of the starting of the pipe along the direction of the water flow is as (8).

$$T_{end,i} - T_s = (T_{start,i} - T_a)e^{-\frac{\lambda_i L_i}{C_m}} \quad \forall l \in HL \quad (8)$$

Where $T_a$ denotes the ambient temperature; $L_i$ denotes the length of the $i$th pipeline; $\lambda_i$ denotes the overall heat transfer coefficient of the $i$th pipe per unit length.

If there is mix at a node, the temperature at the beginning of the pipe flowing out of the node are equal to the mixture temperature of all the hot water in pipes flowing to the node. This is described as (9).

$$(\sum m_{in}) T_{start} = \sum m_{in} T_{end} \quad (9)$$

C. Coupling Elements

The role of coupling elements in the integrated energy system is to achieve different types of energy conversion. The coupling elements involved in this article are CHPs. The working characteristic of CHP is described as (10). The heat power from CHP isn’t allowed to exceed the upper limit and lower limit, which is presented by (11).

$$P_{G,i} = c_m \Phi_{G,i} \quad \forall i \in CHPS \quad (10)$$

$$\Phi_{G,i} \leq \Phi_{G,i}^\text{min} \leq \Phi_{G,i} \leq \Phi_{G,i}^\text{max} \quad (11)$$

Where $c_m$ denotes the power-to-heat ratio.

III. LINEAR OPTIMAL ENERGY FLOW FORMULATION

As shown in Section II, the model of heating network is nonlinear and nonconvex. Assuming the supply temperature at each node and the return temperature at each source node are specified value, in order to linearize the model and combine the hydraulic model and the thermal model together, a new variable called heat quantity $Q = mT$ is built [10]. The topological structures of supply heating network and return heating network are the same. The mass flow rate of pipes at the same location in two networks is the same and the temperature drop coefficients of pipelines $e^{-\frac{\lambda L}{C_m}}$ are also the same. So the supply heating network is only analyzed in the solution process of optimal flow in this paper.

The both sides of the pipe pressure drop equation (5) multiply the $T_{s,start}$ and the equation becomes (12). $T_{s,start}$ denotes the supply temperature of the starting node of a pipe.

$$K_i Q_{start,l} | Q_{start,l} | = T_{s,start,l} \quad \forall l \in HL \quad (12)$$

Denoting $T = T - T_o$ firstly. The both sides of the pipe temperature drop equation (8) multiply the mass flow rate $m$. Meanwhile, molecular and denominator of the index of the temperature drop coefficients of pipelines multiply the mass flow rate $m$. The equation (8) become (13).

$$Q_{end,l} = Q_{start,l} e^{-\frac{\lambda L}{C_m}} \quad \forall l \in HL \quad (13)$$

Based on (7) and (9), the nodal heat quantity balance equation is written as (14).

$$A_{in,i} Q_{in,i} - A_{out,i} Q_{out,i} = \Phi_i T_{o,i} \quad \forall i \in HB \quad (14)$$

Where $A_{in,i}$ denotes the $i$th row of the nodal-pipe incoming matrix; $A_{out,i}$ denotes the $i$th row of the nodal-pipe leaving matrix.

Based on the piecewise linear approximation method, (12)-(13) are formulated linearly as (15)-(19).

$$\sum_{k=1}^{N} (Q_{start,l,k+1} - Q_{start,l,k}) \delta_{l,k} + Q_{start,l} = T_{s,start,l} \quad \forall l \in HL \quad (15)$$

$$Q_{end,l} = \sum_{k=1}^{N} (Q_{end,l,k} - Q_{end,l,k+1}) \delta_{l,k} + Q_{end,l} e^{-\frac{\lambda L}{C_m}} \quad \forall l \in HL \quad (16)$$

$$Q_{start,l} = Q_{start,l,1} + \sum_{k=1}^{N} (Q_{start,l,k+1} - Q_{start,l,k}) \delta_{l,k} \quad \forall l \in HL \quad (17)$$

$$0 \leq \delta_{l,k+1} \leq \delta_{l,k} \leq 1, k = 1, 2, \cdots \cdots , N-1, \forall l \in HL \quad (18)$$

$$0 \leq \delta_{l,k} \leq 1, \eta_{l,k} \in [0,1], k = 1, 2, \cdots \cdots , N, \forall l \in HL \quad (19)$$

Where $\delta_{l,k}$ represents the position on the $k$th segment interval, the range of its value is from 0 to 1; $\eta_{l,k}$ is a binary variable. Equation (18) is used to ensure that the whole segment interval must be filled continuously from left to right without jumping.

Based on the above analysis, the optimal energy flow of combined heating and electricity system is modeled as follows:
the objective function is (1); the constraints of electricity network are (2)-(3); the constraints of heating network are (6) and (14)-(19); the constraints of coupling elements are (10)-(11).

IV. ITERATION METHOD OF OPTIMAL ENERGY FLOW

The mixed integer linear model of optimal energy flow proposed in Section III is based on the assumption that the supply temperature at each load node and the outlet temperature at source node in the heating network are given. But it is difficult to get the true value of these temperature variables under optimal condition directly. In order to achieve successive approximation of truth values of the temperature variables, an iteration method based on the mixed integer linear programming and the calculation of energy flow of the heating network is proposed in this paper. The flowchart of the iteration method is shown in Fig. 2. The steps of the iteration method are as follows.

START
Initialize $T_{\text{load}}=T_{\text{source}}, T_{\text{source}}=T_{\text{load}}$
These Variables denote as $T(0)$, set $k=0$
Solve the optimal energy flow under the pertinent temperature variable is $T^{(0)}$
Solve the heat energy flow according to results of optimal energy flow
Update the pertinent temperature variable $T^{(k+1)}$
Output the solution of the optimal energy flow
END

Fig. 2. Flowchart of the iteration method of optimal energy flow.

Step 1) Assume that the supply temperature at each load node equals to the supply temperature at heat source node and the outlet temperature at source node equals the outlet temperature at load node in heating network. Set $k=0$.

Step 2) Solve the mixed integer linear model of optimal energy flow proposed in Section III under the condition that the pertinent temperature variable is $T^{(k)}$. The mass flow rate of pipes can be calculated by (20) according to the solution of heat quantity. If the values of mass flow rate are not all positive, update the direction of pipelines, solve the optimal energy flow again.

$$m_i = \frac{Q_i}{T_{\text{stat},i}}, \forall i \in \text{HL}$$ (20)

Step 3) According the value of heat power generated from source achieved in Step 2), solve the energy flow of heating network. The method of energy flow calculation of heating network is introduced in detail in [4].

Step 4) According to results of energy flow calculation, the value of the pertinent temperature variable can be achieved as $T^{(k+1)}$. If $\|T^{(k+1)}-T^{(k)}\|_2 < \varepsilon$, output the solution of the optimal energy flow and stop; otherwise, set $k=k+1$, update the value of the pertinent temperature variable and go to Step 2).

V. CASE STUDY

The test system consists of modified IEEE 33 distribution network and modified Barry Island heating network. The schematic diagram of 66 bus combined heating and electricity system is shown as Fig. 3. The detail information of Barry Island heating network is introduced in [4]. Programs are coded using MATLAB R2018b and the mixed integer linear programming is solved by CPLEX.

The calculation result and price of heat power from three CHPs are shown in TABLE I. The calculation result and price of electricity power from three CHPs and external power grid source are shown in TABLE II. The calculation of supply temperature and return temperature of some nodes are shown in Fig. 4. It can be seen that the proposed method is enough accurate because the results of heat energy flow and optimal energy flow based on the mixed integer linear programming are the same finally.

![Fig. 3. Schematic diagram of the test system](image)

TABLE I. RESULTS AND PRICE OF HEAT SOURCE

<table>
<thead>
<tr>
<th>Source</th>
<th>CHP 1</th>
<th>CHP 2</th>
<th>CHP 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (k¥/MW)</td>
<td>112.88</td>
<td>107</td>
<td>112</td>
</tr>
<tr>
<td>Output (MW)</td>
<td>0.7465</td>
<td>0.9820</td>
<td>0.5392</td>
</tr>
</tbody>
</table>

TABLE II. RESULTS AND PRICE OF ELECTRICITY SOURCE

<table>
<thead>
<tr>
<th>Source</th>
<th>CHP 1</th>
<th>CHP 2</th>
<th>CHP 3</th>
<th>external power grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (k¥/MW)</td>
<td>112.88</td>
<td>107</td>
<td>112</td>
<td>500</td>
</tr>
<tr>
<td>Output (MW)</td>
<td>0.8585</td>
<td>1.1293</td>
<td>0.6201</td>
<td>0.3671</td>
</tr>
</tbody>
</table>
The whole calculation process of optimal energy flow converges in four iterations. The total operation cost decreases gradually in the process of iterations as shown in TABLE III. In conclusion, the accuracy and convergence of the iteration method proposed in this paper are good.

TABLE III. TOTAL OPERATION COST IN EACH ITERATION

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total operation</td>
<td>765.9371</td>
<td>745.4824</td>
<td>745.4448</td>
<td>745.4440</td>
</tr>
<tr>
<td>cost (k¥)</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, an iteration method based on the mixed integer linear programming is proposed to solve the optimal energy flow of combined heating and electricity network. In the proposed method, the piecewise linear approximation method is used to convert the nonlinear and nonconvex model of heating network to the mixed integer linear model, in which the initial value of the pertinent temperature variables is given. The true value of these temperature variables under optimal condition are difficult to get directly, thus, an iteration method is proposed to solve the optimal energy flow considering various temperature in heating network. A case study based on a 66 bus combined heating and electricity system demonstrates the effectiveness and accuracy of the proposed method. Main conclusions can be drawn as follows:

1) The proposed iteration method can achieve convergence within 4 iterations, which means it has a good convergent performance;

2) By introduce an energy flow calculation of the heating network in iteration process, the temperature difference between different nodes can be taken into account when optimizing the energy flow of combined heating and electricity system, which makes the results more accuracy.

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REFERENCES